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DISTANCE AND DISCRETE SPACE

Let us say that space is *discrete* if and only if every finite extended region of space is composed of finitely many *atomic* regions of space. I assume here that regions of space are individuals rather than sets of points, and have mereological structure; their parts are all and only their subregions. A region of space is an atomic region if and only if it has no proper parts, i.e., if and only if it is a mereological atom. In what follows, I will simply call atomic regions of space 'atoms'. Let us assume that, necessarily, all atoms are *unextended* regions, i.e., *points* of space.

According to the Weyl Tile argument, no world with discrete space could approximate a world with continuous space because (1) the Pythagorean theorem fails to hold in worlds with discrete space and (2) it is not even approximated as the number of points in a finite region approaches infinity.[1] Consider the following space (asterisks represent points):

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*a   *   *
*b   *   *
*c   *d  *
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Consider the "triangle" made out of a, b, c, and d. On reasonable assumptions about how distance works, the length of the side composed of a, b, and c is *three*. The length of the side composed of c and d is *two*. But the length of the side composed of a and d is also two! So the Pythagorean theorem fails for this space. More importantly, simply enriching the number of points in the space cannot make the Pythagorean theorem even approximately true. Since actual space appears to at least approximate the Pythagorean theorem, this might suggest that we have evidence that space is not discrete.

The Weyl Tile argument presupposes a common and plausible claim about the way that distance works at possible worlds with discrete space. This claim is the *size thesis* (ST), which states that, necessarily, the size of any region of discrete space (regardless of how many dimensions this region has) is a function of the number of points that the region has as parts. If we confine our attention to one-dimensional discrete spaces, we can see that this thesis implies

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that the length of any line-segment is determined entirely by the number of points that it has as parts. I will argue that ST is false.[2]

The reader should be warned that the argument I will present is a *metaphysical* argument against ST, not a mathematical argument. Moreover, it does not purport to establish that there are no worlds in which the size of a region is a function of the number of parts of the region. For it is consistent with the falsity of ST that there are such worlds.

My first premise is that there are possible worlds according to which space is continuous and distance is *perfectly natural, external* and *pervasive*. On naturalness: I assume that some properties and relations are more natural than other properties and relations. There are three roles that naturalness is invoked to play. First, the naturalness of a property determines the degree of objective similarity that it confers on those entities that exemplify it. Second, the pattern of instantiation of the perfectly natural properties and relations determines the pattern of instantiation of every other qualitative property and relation. Third, objects are intrinsic duplicates if and only if there is a one-to-one correspondence between their parts that preserves perfectly natural properties and relations; a property is intrinsic only if it never differs between duplicates (Lewis 1986, pp. 59–69). Strictly speaking, I hold that it is the determinates of the distance relation, i.e., the various relations of the form x is n units from y, that are perfectly natural.

On externality: external relations do not supervene on the intrinsic properties of their relata; however, they do supervene on the intrinsic properties of the fusion of the relata. External relations should be contrasted with *extrinsic* relations, which do not even supervene on the qualitative character of the fusion of their relata (Lewis 1986, pp. 62–63). An example of an extrinsic relation is ownership. Ownership does not supervene simply on the qualitative character of the owner and the owned; instead, it supervenes on that character taken along with the various social facts that accompany it. That distances are external relations follows from the claim that they are perfectly natural so, strictly speaking, this is not an additional condition.[3]

On pervasiveness: a determinable relation R is pervasive if and only if whenever some determinate of R directly relates a to b and some determinate of R directly relates b to c, then some determinate of R directly relates a to c. Pervasiveness must not be confused with transitivity. Distance is a determinable relation: its determinates are the various relations of the form x is n units from y. If distance is pervasive, then if a

bears some determinate distance relation to *b*, and *b* bears some determinate distance relation to *c*, it follows that *a* bears some determinate distance relation to *c*, where this latter relation is also direct. However, the distance relation that *a* bears to *c* need not be – and almost certainly will not be – the relation that *a* bears to *b* or that *b* bears to *c*.

These three assumptions characterize the standard view about how distance relations behave in a continuous space.[4] I do not assume that every possible world is such that the distance relations at that world are perfectly natural, external and pervasive.[5] But surely there are possible worlds according to which space is continuous and distance has these features. Let us attend to one of these worlds; for ease of exposition, I will pretend that one such world is the actual world.

My second premise is the claim that the proper parts of space, which are its subregions, are contingently existing objects. I suspect that most philosophers who believe in the possibility of discrete space will endorse this premise. For consider a possible world *w* according to which space is finite and discrete. Given these features, this possible world represents space as having a finite number of parts. However, in the actual world, space has a non-denumerably infinite number of parts. In *w*, some (or all) of those parts must be unaccounted for.[6] This suffices to show that some proper parts of space exist contingently. But once we accept that some parts exist contingently, we should think that each part is a contingent being. It would be extremely arbitrary to hold that some proper parts of space exist contingently while others enjoy necessary existence.[7]

My third premise is a Humean principle of recombination. More accurately, it is a principle of subtraction applied to contingently existing objects. In stating the principle, I invoke the following mereological concept:

> *x overlaps y* if and only if there is some *z* such that *z* is a part of *x* and *z* is a part of *y*.

The principle is as follows:

> (SUB): Let *w* be a possible world and let *x* be a contingently existing object that exists at *w*. Then there is a possible world *w*2 such that (i) *x* exists at *w*2, (ii) every contingent object that exists at *w*2 overlaps *x*, and (iii) for any intrinsic property *P*, *x* has *P* at *w* if and only if *x* has *P* at *w*2.[8]

SUB is based on the familiar Humean idea that there are no necessary connections between distinct (read: non-overlapping) existents.[9]

If A and B are completely distinct contingent objects, then how A is intrinsically, or whether A even exists, should be metaphysically independent of the nature and existence of B.[10]

Given SUB, one can "subtract" contingently existing objects from the actual world while preserving the external determinate distance relations that obtain amongst those entities that are remaining.

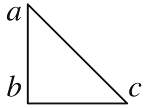
Each of these premises is plausible. However, their conjunction implies that ST is false. To see this, suppose that space is continuous, and consider three collinear points m, n, and p, with m and p one meter apart, and n 0.4 meters from m and 0.6 meters from p. Let us call the discontinuous region of space composed of m, n and p, "Bitty". Given premise (1), the distance relations obtaining between m and n, n and p, and m and p, are external. As mentioned earlier, external relations supervene on the intrinsic properties of the fusion of their relata. This implies that the distance relations obtaining between m, n, and p supervene on the intrinsic properties of Bitty. Let us call the set of intrinsic properties had by Bitty "I".

Given premise (2), each of m, n, p, and Bitty is a contingently existing object. Given premise (3), there is a possible world w at which (i) Bitty exists, (ii) every contingent object that exists at w overlaps Bitty, (iii) Bitty instantiates every intrinsic property in I, and (iv) every intrinsic property instantiated by Bitty is in I.

m, n, and p exist at w. In w, space is discrete, because there are only finitely many spatial atoms (in w, there are only three spatial atoms). Moreover, since the distance relations obtaining between m, n, and p supervene on the intrinsic properties of Bitty, m, n, and p bear the same distance relations to each other as they do in the actual world, despite the non-existence of the space in which they are embedded in the actual world.

If ST is true, then, at world w, the distance from m to n equals the distance from n to p, since the number of spatial atoms between m and n is equal to the number of spatial atoms between n and p (both numbers are zero). Nevertheless, the distance from n to p is greater than the distance from m to n. In w, the distance between two spatial atoms is not simply a function of the number of atoms lying between them, contrary to what is claimed by ST. Given our Humean argument, ST is false.

Recall that, according to the Weyl Tile argument, no world with discrete space could approximate a world with continuous space because the Pythagorean theorem fails to hold in worlds with discrete space. If the principles appealed to in the previous section are true, the Weyl Tile argument fails. In fact, if we accept these principles, then we have a very easy recipe for constructing worlds with discrete space that obey the Pythagorean Theorem. Begin with a world $w1$ at which distance is pervasive and perfectly natural. Consider a triangle-shaped region in that space:



Next, via SUB, delete every point in this space save a, b, and c, but preserve the various distance relations that obtain between these three points. (Since the open-ended line segment composed of the points between a and b does not overlap a or b, and similarly for the other two line segments, SUB can be applied in this way.) This takes us to this world, $w2$:

a^*

b^* $\qquad$ c^*

At $w2$, space is discrete. But at this world, the Pythagorean theorem holds, since the distance between a and b at world $w1$ equals the distance between a and b at $w2$, and so forth for the pairs $\langle a, c\rangle$ and $\langle b, c\rangle$. So, once we reject ST, we can reject the Weyl Tile argument as well.

In retrospect, perhaps this result is not terribly surprising. We already knew that in a continuous space, the distances between points are not determined in accordance with an unrestricted version of the Size Thesis. But in worlds with discrete space, it seems natural to correlate the distances between points with the number of points between them. Perhaps this is what initially tempts us to the Size Thesis. What I have shown is that the denial of this assumption follows from plausible principles.[11]

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NOTES

[1] A clean statement of the Weyl Tile Argument can be found in Salmon (1980, pp. 62–66).

[2] I am heavily indebted to Bricker (1993) for what follows. The argument developed here makes use of premises that are defended in that article.
[3] See Lewis (1986, pp. 61–62).
[4] David Lewis assumes that distance, or some relation analogous to distance, has these features at every possible world Lewis (1986, pp. 75–76).
[5] Phillip Bricker argues in Bricker (1993) that if space (or spacetime) is as represented in General Relativity, distance relations (or interval relations), such as *x is five meters from y*, are not external.
[6] One could, of course, save the claim that all of the actually existing regions exist at w by claiming that some distinct regions at the actual world are identical at w. This strikes me as a high price to pay to save the claim that spatial regions exist necessarily.
[7] It of course does not follow from the claim that every proper part of space is a contingently existing being that there is a possible world in which there is no space. It may well be necessary that space exists at every world. But this claim is consistent with my second premise.
[8] I do not wish to commit myself here to any theory about how the transworld identity of regions "works". Perhaps possible worlds represent that an object exists at a world by "containing" the object itself. Or perhaps possible worlds represent that an object exists at a world by "containing" a counterpart of that object. I take no stand on this issue here.
[9] Armstrong seems to endorse a version of this principle in Armstrong (1989, pp. 61–65). Lewis endorses a stronger principle in Lewis (1986, pp. 86–92) that implies SUB.
[10] How A is intrinsically, or whether A even exists, may be *causally* dependent on the nature and existence of B. But this is of course consistent with SUB.
[11] I thank Jake Bridge and Phillip Bricker for comments on an earlier draft of this paper.

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